Undergraduate Mathematics Study Groups: What Mathematical Talk Actually Takes Place?

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This paper reports on a study that investigated the nature of the students' mathematical talk in an undergraduate mathematics study group. Study groups to support the learning of first year mathematics students are encouraged by mathematics educators. From nine recorded sessions, a session with a high quantity of mathematical talk and unfamiliar topics was chosen as a case study. The students used several different interactions but they developed mostly low level cognitive conversations. Four proposed causes limiting the cognitive level of the student's mathematical talk are the lack of prior preparation, avoidance of high level cognitive questions and the inability to recognise and then develop opportunities.

An increase in the range of first year students' mathematical prior knowledge (Rylands & Coady, 2009) prompted the mathematics educators at a Queensland university to encourage students to form study groups (Belward & Balatti, 2012). Participation in study groups has been a common factor in the success of students in mathematics (Fullilove & Treisman, 1990). While the benefits from study groups accrue when the students talk, the research on the nature of the student talk in study groups is limited (Christian & Talanquer, 2012). The paucity of research into the nature of the student talk in tertiary mathematics study groups means that mathematics educators' understanding of how students' mathematical talk supports their learning and how they can help enhance such talk is limited.

This research used a case study methodology to explore the nature of the students' mathematical talk in a study group and how the students' talk appeared to support the learning of mathematics. The research was a part of a larger project that explored the students' talk in mathematics study groups including the patterns of student talk and the nature of the mathematical talk.

Literature Review

Christian and Talanquer (2012) defined study groups as self-selected, self-directed and self-regulated small groups of students who meet for the purposes of helping one another learn . The study group in this research was self-selected in a tutorial and the participants made mutual decisions regarding their meeting time and place and the work they completed in the study session.

The research concerning mathematics study groups is limited and most has relied on self-reports rather than observations (Christian & Talanquer, 2012). Fullilove and Treisman (1990) noted the connection between the use of study groups and the academic success of tertiary students studying mathematics. Research in the undergraduate mathematics context includes that of Lazar (1995) who explored students' purposes for joining study groups and that of Sandoval-Lucero, Blasius, Klingsmith, and Waite (2012) who explored students' perceptions of study groups. Both studies reported that students in the study groups stated

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that discussions with their peers were integral to supporting their learning and developing their understanding (Lazar, 1995; Sandoval-Lucero et al., 2012).

Learning in study groups is supported by both cognitivist and constructivist learning theories (McNair, 2000; Webb, 2009). On the “intramental plane” (McNair, 2000, p. 198) cognitive elaboration takes place when the student prepares to disseminate an idea (Slavin, 2010; Webb, 2009) whereas socio-cognitive constructivism is the cognitive restructuring of knowledge following episodes of socio-cognitive conflict (Webb, 2009; Webb & Mastergeorge, 2003). In contrast, socio-cultural constructivism is an “intermental process” (McNair, 2000, p. 198) involving the shared construction of understanding among individuals in a social setting (Webb, 2009; Webb & Mastergeorge, 2003). In the study group context, the learning that these theories explain is dependent on student talk.

In this study, the focus was on the students’ mathematical talk. Student talk has been defined as “mathematical” because of the nature of the language used (Sfard, Nesher, Streefland, Cobb, & Mason, 1998) or because of the purpose of the talk (McNair, 2000). Nesher in Sfard et al. (1998) described mathematical talk as the use of mathematical language rather than the use of everyday language to discuss mathematics. McNair (2000) declared that “mathematical discussion should have a mathematical subject and a mathematical purpose” where a mathematical subject is a field of mathematical study such as number or geometry and a mathematical purpose is a reason that requires the conduct of mathematical operations (McNair, 2000).

Research into student-student interactions has explored the nature of student talk in study groups (Christian & Talanquer, 2012; Goos, Galbraith, & Renshaw, 1996; Haller, Gallagher, Weldon, & Felder, 2000). In the three studies, the student talk was described according to the type of interaction: Goos et al. (1996) described the interactions according to the amount of sharing of information, the extent of direction giving and the use of explanations; Christian and Talanquer (2012) used knowledge symmetries; and Haller et al. (2000) used a combination of knowledge symmetries and conversation analysis. Knowledge symmetries describe the regulation of the talk with asymmetrical conversations being when superior student/s dominate the bulk of the conversation and dictate the turn-taking while symmetrical conversations have no superior students (Christian & Talanquer, 2012).

From the three studies four different interactions were identified: peer tutoring (Christian & Talanquer, 2012; Goos et al., 1996) or transfer of knowledge (Haller et al., 2000); teaching (Christian & Talanquer, 2012); collaboration (Goos et al., 1996; Haller et al., 2000) or co-construction (Christian & Talanquer, 2012); and parallel activity (Goos et al., 1996). Tutoring interactions are asymmetric conversations where a single student with superior competence controls the conversation and provides explanations for the other students. Teaching interactions differ to tutoring in that the content and format of the support is determined by the more knowledgeable student. Co-construction interactions are symmetric conversations where students share ideas and explanations to develop understanding together. In contrast, parallel activity has very little sharing of ideas with the students working independently.

Christian and Talanquer (2012) in their study of student talk in a chemistry context also coded the cognitive level of the student talk by applying a version of Bloom’s Taxonomy. The MATH taxonomy was similarly developed by Smith et al. (1996) to evaluate the cognitive demands of mathematical assessment questions. Within the MATH Taxonomy, most definitions describe low or medium level cognitive activity as the application of procedural knowledge and medium or high level cognitive activity as the explication of conceptual knowledge. However, as the distinction between procedural and conceptual knowledge is unclear (Engelbrecht, Bergsten, & Kågesten, 2009), the context of the talk, as listed in the adaption of the MATH Taxonomy (see Table 1), better reflects the cognitive level of the student talk than the categories of procedural and conceptual knowledge.

Table 1.
Adaption of the MATH Taxonomy

Low cognitive activity	
Factual knowledge	Reproducing previously learned information
Comprehension	Using simple definitions, recognising and understanding mathematical symbols
Routine procedures	Using simple equations or carrying out basic manipulation
Medium cognitive activity	
Information transfer	Transforming information from verbal to number or vice versa, explaining relationships and processes
Application to new situations	Modelling in real life, choosing and applying mathematical concepts, using either known procedures in unfamiliar situations or using unknown procedures
High cognitive activity	
Justifying and interpreting	Proving a theorem, recognising errors in reasoning and limitations, discussing examples and counter-examples and recognising unstated assumptions
Implications, conjectures and comparisons	Making and proving conjectures, comparing algorithms, deducing implications of a result
Evaluation	Making judgements, selecting for relevance, arguing merit and organising or creating information

Adapted from Smith et al. (1996)

Methodology

Context

The first year mathematics cohort organised themselves into student-selected study groups of between three and six students in the tutorials in the first week of semester. The study groups were given the general instructions to meet weekly and to focus on completing questions provided in the lecture notes. Study group work was not assessed and no marks were given for study group participation. A total of nine groups responded to a request for volunteers to participate in the study. Extenuating circumstances meant that only six different groups were able to participate. The final data comprised a total of nine videoed sessions with an average duration of one hour each. All were coded for duration of mathematical talk before selecting the session for the case study reported here.

The case selected had one of the highest amounts of mathematical talk (54%). The size of the group (three participants) was typical of most study sessions. Finally, the topics of the mathematical talk in the sessions were proportionality and combinations with which the students had some limited prior knowledge. The participants were given the pseudonyms of Fred, John and Frank. In an early semester diagnostic test of basic arithmetic and algebra the members of this group achieved very high or high results.

Method

In a small comfortable meeting room two cameras were set up angled to capture the students' workspaces but not their faces. At the end of the session the students' written work was photocopied to enable connections between the student talk and the students' written text.

The audio recording of the nine sessions were coded into segments of on-task talk, off-task talk or silence. On-task talk comprised any talk that was related to doing mathematics. Time in excess of ten seconds without talk was coded as silent. The on-task talk segments were re-coded as mathematical or non-mathematical talk using the description of McNair (2000). The mathematical talk fragments were then transcribed using a process adapted from Jefferson Transcription Notation (Tuckman, 1999). The transcript was broken into episodes of talk comprising passages of talk that was all linked to the same purpose. An episode ended when the purpose changed, was fulfilled or the talk ceased. Duration of the episodes ranged from 11 seconds to four minutes and 49 seconds.

The student talk in each episode was coded according to the type of interaction: co-construction; tutoring; teaching; and simple. Simple interactions have a mathematical subject and a mathematical purpose, but no symmetry or other features that would classify it as a co-construction, peer tutoring or teaching interaction. Parallel activity interactions could not be coded as the key indicator is extended periods of silence which are not evident in the data. The cognitive level of the student talk was coded using the adaption of the MATH Taxonomy (see Table 1).

Findings

The case study produced 63 episodes. Thirty five percent of the episodes were co-construction, 32% were tutoring and 33% were simple interactions. However, the amount of talk each interaction comprised varied from 50% for co-construction and 35% for tutoring down to 15% for simple interactions. No episodes of teaching were coded. Coding of the cognitive level of the episodes found that all of the simple interactions, 86% of the co-construction interactions were of a low cognitive level and 60% of the tutoring interactions were of a low cognitive level. There were no high cognitive level episodes. Below are excerpts from the episodes that illustrate different interactions and cognitive levels.

The first excerpt is an example of a low cognitive level, tutoring interaction which took place midway through the session. Fred and Frank were working on a question related to combinations. The clear power asymmetry, with Frank taking the superior role and Fred the learner role, indicates a tutoring interaction and the short responses are indicative of a low cognitive level. Unlike typical tutoring interactions, this interaction is one in which the learner is doing most of the explaining and receiving confirmation from the superior participant.

Fred:	In how many ways can a team of four be chosen from seven players? So it's seven at the top?

Frank: Yep.
Fred: And the bottom's four?
Frank: So, then you do a combination of that.

In contrast, the excerpt below is an example of a medium cognitive level, co-construction interaction. The students were to show that number of combinations of reading five books from a list of 20 when three books were compulsory was 136. The student's equal contributions and the short and often interrupted statements are typical of a co-construction interaction (Note: the square brackets indicate simultaneous talk). The shared understanding used in translating the information from the written problem to a mathematical format indicates a medium cognitive level.

John: So it would be like seventeen choose five. Is that right?
Fred: I was thinking maybe … three must be read.
John: So it's seventeen choose two? … 'cause three always have to be read so you [take three books …
Fred: Five altogether must be read.
John: Yeah, you take three books away from the possible the two- … the possible books …
Fred: So there are two more you have to read.
John: And yeah, there's two left so it would just be seventeen choose two

The ability of the students to develop a shared understanding of the processes and concepts was an important factor in their ability to complete the tasks. In the first excerpt Fred uses everyday language to explain his translation of the information from the written question to mathematical representation, ${}^{7}C_{4}$. While the translation is procedurally correct, no explanation drawing on the context is provided. In contrast, the second excerpt shows an improvement in the students' ability to translate the information due to the shared understanding of the language, "seventeen choose two". This improved talk made explicit connections with the context. However, the students were unable to develop a shared understanding of the difference between combination and permutation. The excerpt below illustrates the student's attempt to develop an understanding of the difference between combinations and permutations earlier in the session.

Fred: So, combination or permutation?
Frank: Ahhh, permutation because order doesn't matter.
Fred: No, in permutation it matters.
John: Yeah, permutation is when …
Fred: Order matters with permutations. P is for position.
Frank: Yeah, yeah. No, it's not. Okay, combination.
Fred: This is how I remember. I read it on a website yesterday; permutation position.
Frank: Ahhh, okay.
Fred: That's how I remember it.

The students' talk was limited to statements of a low cognitive level. An example is Fred's comment, "permutation position", is a mnemonic and has no conceptual basis. The consequence of a limited conceptual understanding was apparent in the next excerpt of dialogue concerning a subsequent question which shows that the students were again unable to ascertain the nature of the new question involved combinations or permutations.

Fred: So is this combination or permutation? I'm all confused. Combination is it?
Frank: I got forty five …
Fred: Okay, then I did permutation. Dang it!

The students failed to recognise that their conceptual understanding was insufficient. For example, after getting the wrong answer the student changed their calculation to suit a combination, but he did not discuss why he was incorrect. The “combination or permutation” dilemma occurred eight times in the session, thus highlighting the need for the students to engage in a high cognitive level of talk to develop their conceptual understanding.

When opportunities for high level cognitive talk did arise the students were unable to generate high level cognitive discussions. Earlier in the session a proportionality question on compound interest provided an opportunity for a conceptual discussion when the student’s explored the necessity of the constant ‘k’. As shown in the excerpt below, Fred’s failure to explain and justify his belief the ‘k’ is necessary is an example of the student’s struggle to generate high level cognitive discussions.

Frank:	I don’t understand why you need k.
Fred:	You do, that’s the starting value. I did accounting before ok. ((Laughs))
Frank:	But you can work out- … ok.
Fred:	((Laughs)) I don’t know, that’s what I do. Ok?
John:	I didn’t use k for umm (question) one … two, or I haven’t used k yet at all but
Fred:	Yeah but you gotta use … you gotta find k.

The student’s decision not to engage in challenging or non-routine questions limited their opportunities for development of conceptual understanding. The only two questions in the exercise that were abstract questions dependent on conceptual understanding of the notation were not attempted, nor even mentioned in the study session.

A conceptual understanding is essential for the students to be able to take a logical approach to solving the questions. The following excerpt of a simple interaction illustrates how an incomplete conceptual understanding limited the student to a guess-and-check approach to solve the question. Fred is attempting the second part of a question and in the first line he shares the numbers he has decided are relevant based on the results of the first part of the question. Fred’s decision is based on an arithmetic connection unrelated to the question and he admits that his method of choice is trial and error.

Fred:	Something with thirty five … seven … four … how about eleven? I’m gonna try eleven seven … seven plus four is eleven
All:	((Laughs))
Fred:	Trial and error?

As illustrated by the excerpts in this section, the student’s employed co-construction, tutoring and simple interactions but the cognitive level of the talk was generally low. There was no student talk of a high cognitive level because: the students did not recognise the need to improve their conceptual understanding; the students were unable to develop high level cognitive talk; and the students avoided the abstract questions which required high level conceptual understanding. However, the students did develop shared understandings of some mathematical language used to describe the notation.

Discussion and Conclusions

The research explored the nature of the students’ mathematical talk from a single case study. The session for the case study was selected for its high quantity of mathematical talk. The interactions employed by the students, the language used by the students and the cognitive level of the students’ mathematical talk were explored.

Two thirds of the students’ mathematical talk was evenly divided between co-construction and tutoring interactions. The remaining third of the time the students engaged in simple interactions. Some of this time could have included parallel activity. Parallel activity interactions are mostly silent but any talk that does occur comprises simple interactions for checking or clarifying information (Goos et al., 1996).

In this study, there were no teaching episodes. Teaching interactions only occur when individuals in the study group had a far superior knowledge than their peers (Christian & Talanquer, 2012). The absence of substantial knowledge could explain the absence of teaching interactions in the case study.

The tutoring interactions comprised student talk that was limited to low and medium cognitive levels. Because the three students began the session with a sparse recollection of the topics, the tutor's understanding was generally only slightly superior to the tutee. Consequently, much of the tutor's talk comprised factual responses without explanation which limited the cognitive level of the student talk. The learning gains of both the tutor and the tutee are related to the cognitive level of the student talk in the tutoring interactions (Webb, 2009).

The student talk in the co-construction interactions was primarily of a low cognitive level. A contributing factor may have been the procedural questions that the students chose to complete. Christian and Talanquer (2012) noted that working on procedural questions generally produced low level cognitive talk. In contrast, when the students worked on written problem solving questions that required them to transfer information and apply it to new situations medium level cognitive talk was evident.

The students developed a shared understanding including the written and verbal communication of the specific notation and connections with the context of the problems. Nesher in Sfard et al. (1998) commented on the difficulty of expressing mathematical situations using everyday language and the need to use mathematical language. The use of shared language is critical to the student's ability to construct solutions to new questions (Christian & Talanquer, 2012) and enabled them to translate and complete the written problem solving questions later in the session.

Opportunities for talk of a high cognitive level were evident in the session. For example, the students repeatedly addressed the "combination or permutation" nature of the problem solving questions. However, the students' capacity to exploit these opportunities was limited by their ability to recognise opportunities when they arose and even when they did, they were unable to generate the high level cognitive discussions required. The students' opportunities for high level cognitive talk were further reduced by their avoidance of higher level cognitive questions (Christian & Talanquer, 2012).

In conclusion, the students' mathematical talk produced co-construction, tutoring and simple interactions in roughly equal quantities. While the students' mathematical talk was primarily of a low cognitive level, the students did develop a shared understanding and produced some talk of a medium cognitive level. The absence of substantial knowledge among the students had a negative effect on the quantity of medium and high level cognitive talk thus limiting the students' opportunities for conceptual learning.

The single case cannot be taken as representative of the study group session generally or even the total data set in this study. However, it does illustrate the limitations of having the students work in study groups without any support. Further research directions include developing interventions that enhance the cognitive quality of the students' talk by emphasizing prior preparation, enabling the students to identify the need for further learning and improving their capacity to engage in social interactions that produce knowledge.

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